

How much redistribution is possible through commodity taxes?

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ABSTRACT

This paper aims to quantify the redistribution which can be attained as the outcome of indirect taxation. This question is of crucial relevance to many countries which are constrained to depend primarily on indirect taxes. I assess the maximum possible redistribution by imposing the most redistributive indirect taxation. The public budget is balanced, and the real income gain to the worst-off is taken as the metric of redistribution. Ceilings on this metric are first obtained within a general set-up, and then in a number of specific models. The resulting redistribution is found to be meager. As this is the outcome of the most redistributive taxation, redistribution from any real-world indirect taxation will be even smaller.

HOW MUCH REDISTRIBUTION IS POSSIBLE THROUGH COMMODITY TAXES?

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This paper aims to quantify the redistribution which can be attained as the outcome of indirect taxation. This question is of crucial relevance to many countries which are constrained to depend primarily on indirect taxes. I assess the maximum possible redistribution by imposing the most redistributive indirect taxation. The public budget is balanced, and the real income gain to the worst-off is taken as the metric of redistribution. Ceilings on this metric are first obtained within a general set-up, and then in a number of specific models. The resulting redistribution is found to be meager. As this is the outcome of the most redistributive taxation, redistribution from any real-world indirect taxation will be even smaller.

1. Introduction

The purpose of this paper is to find out how much redistribution is possible when the government is restricted to using commodity taxes only. The reason for addressing this issue is quite strong. Most developing countries are constrained to use indirect taxes, like sales and excise taxes, as their primary policy instruments [Cnossen (1977)]. There are countries, like India, in which as much as 80 percent of the tax revenue and as much as 15 percent of GDP is collected through indirect taxes. This constraint on the available instruments is dictated by the institutional facts of these countries, and it is grossly unrealistic to assume that *any* other tax instrument, like poll tax or income tax, is widely enforceable in these countries. Therefore, a taxation-based redistribution program must rely heavily on indirect taxes. Given this situation, if it is found that only meager redistribution is possible through indirect taxes, then the harsh conclusion would emerge that it is futile to remedy the existing inequities with the help of conventional tools. It is therefore crucial for policy-makers to assess the redistributive capability of indirect taxes.

This important question, however, has not been posed at all in the existing literature on indirect taxation [Atkinson and Stiglitz (1980), Mirrlees (1975,

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1976), Feldstein (1972)]. The focus of the existing literature has been on studying the structure of indirect taxes for a specified social welfare function. These studies of equity-sensitive taxation, however, remain quite incomplete unless one knows what the final outcome of such taxes is, i.e. how much equity *actually* can be achieved. The present paper thus removes a significant gap in our understanding of indirect taxation by analysing this new and relevant question.

The logic of this exercise is as follows. I exaggerate the possibility of redistribution whenever it is convenient to do so. The resulting redistribution is then studied, because this redistribution is as an upper limit of what it would have been if there was no exaggeration. For instance, the welfare of the worst-off individual is maximized using indirect taxation. I call the resulting tax program the 'Ramsey–Rawls program'. The redistribution arising out of a tax program is assessed by measuring the proportional increase in the real income of the worst-off individual. This is explained below. It is obvious that any real-world social welfare function will possess substantially less aversion to inequality than the Rawls case. Consequently, any real-world indirect tax program will be significantly less redistributive than the Ramsey–Rawls program. The redistribution under the Ramsey–Rawls program is, therefore, the maximum limit of redistribution through indirect taxes. In calculating the above taxes and the metric, the public budget is balanced, because my interest here is in examining the purely redistributive aspects of indirect taxation. It is also worth stating here that in many of the situations to be encountered in this paper, the limit on redistribution is obtained solely from the public budget constraint.

The paper is organized as follows. The redistributive metric is explained in section 2. In the latter part of section 2 and in section 3 expressions for this metric are obtained and interpreted for the general case. Specific models of the economy are then considered in sections 4 and 5 in which the redistributive metric is obtained explicitly for a number of cases. While these models do entail simplification, they nevertheless provide unambiguous and useful insights on the question of our interest. In section 6, I discuss the assumptions of the present exercise, and some possible extensions. Finally, a summary is presented in section 7.

2. Metric and a limit on redistribution

There are $h = 1, \dots, H$ individuals. For individual h , $v^h(q, m^h)$ is the indirect utility function, x^h is the row vector of normal goods consumed, and m^h is fixed labor plus non-labor income. p and q are row vectors of pre- and post-tax prices; p is fixed and $q = p + t$, where t is the row vector of taxes. $T^h = x^h t'$ is the tax paid by h . A summation without an index means that the sum is

being taken over h . The public revenue constraint is

$$\sum T^h = 0 \quad (1)$$

Obviously, some taxes will be negative; negative taxes are subsidies. It follows that the uniform taxation is irrelevant here, because it implies zero taxes. Let l be the worst-off household in both the pre-tax and the post-tax price regimes. This assumption and the identification of l are discussed below.

The metric of redistribution measures the proportional increase in the real income of the worst-off individual due to the change in the price regime from p to q . This is motivated as follows. Let I^l be a hypothetical payment to the individual l at the pre-tax price p such that l is indifferent between receiving I^l and changing over to the post-tax price regime q , then

$$I^l = E^l[p, v^l(q, m^l)] - m^l, \quad (2)$$

where E^l is the expenditure function. I^l/m^l is therefore the proportional increase in the real income due to the tax program, and this is the metric of redistribution used in this exercise. It is obvious that the feasibility of direct transfers is entirely unnecessary for the calculation of the hypothetical payment I^l .

The metric can also be interpreted in relation to the 'welfare index' [Deaton and Muellbauer (1980, p. 180)], which is also called the 'true index of real income' [Theil (1980, p. 24)]. The welfare index W^l is defined as $W^l = E^l[p, v^l(q, m^l)]/E^l[p, v^l(p, m^l)]$. The numerator is the expenditure needed at pre-tax prices to reach the post-tax welfare. The denominator is the money income m^l . Substitution of this in (2) and in the definition of W^l gives $I^l/m^l = W^l - 1$. The metric I^l/m^l therefore is the increase in the welfare index of the worst-off, because the welfare index is one in the pre-tax regime. Needless to say, $I^l/m^l > 0$ if the welfare of the worst-off has improved due to taxation.

A simple relation is quickly established between the metric I^l/m^l and the tax payment by individual l . From the concavity of E^l in prices, $E^l[q, v^l] + \nabla_q E^l[q, v^l](p' - q) \geq E^l[p, v^l]$. Substituting this in (2), and using $\nabla_q E^l = x^l$, and $E^l[q, v^l] = m^l$, one obtains:

$$-T^l/m^l \geq I^l/m^l. \quad (3)$$

That is, the negative of the tax payment is an upper ceiling of the real income gain to an individual from the tax program. The intuition is simple. If the welfare of l has improved, then he must receive a net tax subsidy on his total purchases, i.e. $-T^l/m^l > 0$. A part of this subsidy goes to meet the deadweight loss, and the remainder is the increase in his real income. To

establish an upper limit of the metric of redistribution, therefore, it is sufficient to calculate $-T^l/m^l$, if it is convenient to do so. This procedure will often be followed in the present exercise.

With the simple set-up described above, we are already in a position to establish a most general limit on the possible redistribution. Let $\theta_i = t_i/q_i$ be the tax rate. As $q_i = p_i + t_i$, and $q_i > 0$, it follows that $\theta_i < 1$ for any finite tax level. Define $w_i^h = x_i^h q_i/m^h$ as the budget share of individual h for good i . $x_i = \sum x_i^h/H$ is the economy-wide average consumption of good i , and $w_i = x_i q_i/m$ is the average budget share, where $m = \sum m^h/H$ is the average income. It is assumed here that all w_i^h are strictly positive. The possibility of a zero budget share is discussed below. The public revenue constraint (1) becomes

$$\sum_i \theta_i w_i = 0. \quad (4)$$

Divide the index set of commodities into two subsets, J and K . $i \in J$ if $\theta_i \geq 0$, and $i \in K$ if $\theta_i < 0$. If no index set is indicated, as in (4), it means that i ranges over all the commodities. Eq. (4) yields

$$\sum_{i \in J} \theta_i w_i = - \sum_{i \in K} \theta_i w_i. \quad (5)$$

Now, $\sum_{i \in J} w_i > \sum_{i \in J} \theta_i w_i$, because $1 > \theta_i \geq 0$, if $i \in J$. Also, $1 > \sum_{i \in J} w_i$, by definition. Thus, $1 > - \sum_{i \in K} \theta_i w_i$, from (5). Divide both sides by $\min_j \{w_j\}$. As $w_i/\min_j \{w_j\} \geq 1$ for all i , it follows that

$$- \sum_{i \in K} \theta_i w_i / \min_j \{w_j\} \geq - \sum_{i \in K} \theta_i,$$

because $-\theta_i > 0$, if $i \in K$. This gives

$$1/\min_j \{w_j\} > - \sum_{i \in K} \theta_i. \quad (6)$$

Using the same definitions as above,

$$-T^l/m^l = - \sum_i \theta_i w_i^l = - \sum_{i \in J} \theta_i w_i^l - \sum_{i \in K} \theta_i w_i^l.$$

The first part in the right-hand side is clearly negative. Therefore, $-T^l/m^l < - \sum_{i \in K} \theta_i w_i^l$. On the right-hand side of the previous expression, $-\theta_i > 0$, and $w_i^l \leq \max_j \{w_j^l\}$. It follows that $-T^l/m^l < [- \sum_{i \in K} \theta_i] \max_j \{w_j^l\}$. Substituting (6) in this:

$$-T^l/m^l < \max_j \{w_j^l\} / \min_j \{w_j\}. \quad (7)$$

We have obtained a ceiling on the possible redistribution entirely in terms of the budget shares. The proportional real income gain to the worst-off is always smaller than the ratio of the maximum budget share of the worst-off, and the minimum average budget share in the economy. Of course, this limit is overly exaggerated because I have used several strict inequalities in deriving (7). The novelty of this result, however, is that it is derived solely from the public revenue constraint; it does not depend on the actual tax structure, and it does not place any restriction whatsoever on the preferences or on the income distribution within the economy. Moreover, economists know the possible range of budget shares much better than anything else. By substituting plausible values of the budget share in (7), therefore, one can obtain the limit, exaggerated though it is, of how much redistribution can occur.

The result, (7), can also be interpreted if the income distribution is extremely skewed. Let r be the richest individual. If $m^r \rightarrow \infty$, then, $w_i \rightarrow w_i^r$ from the definition. Hence, the denominator in (7) is replaced by $\min_j \{w_j^r\}$. The right-hand side of (7) clearly remains finite. Now, let the largest budget share of the worst-off be 80 percent, and the smallest budget share of the richest be 20 percent. Presumably, both these shares correspond to the necessity good. Then the real income of the worst-off can not increase by more than four times, even though infinite income difference exists!

The main point at this stage is not to establish the narrowest limit. One needs to put more structure into the problem for doing that, and this I do in the subsequent sections. The focus in this section was to demonstrate that the redistribution through indirect taxation faces quite basic limitations, which arise solely from the public budget constraint. These limitations hold — in fact they can only become tighter — regardless of the social welfare function that might be used to obtain the tax structure.

3. Taxes and redistribution

The tax rules can be quickly obtained from

$$\max_t v^l(q, m^l) + \xi \sum T^h. \quad (8)$$

For individual h , let $\alpha^h = \partial v^h / \partial m^h$ be the marginal utility of income and S^h be the Slutsky matrix. $\partial T^h / \partial m^h = (\partial x^h / \partial m^h) t^h$ is the marginal propensity to pay taxes out of income. Let $B^h = 1 - \partial T^h / \partial m^h$. B^h is the net gain to h from a unit addition to income, after correcting for marginal tax payment. The first order conditions of (8), using the above expressions, are: $-\alpha^l x^l / \xi + \sum t^h S^h + \sum B^h x^h = 0$. Post-multiplication of this by q^l , and the use of homogeneity restriction, $S^h q^l = 0$, and the budget identity, $x^h q^l = m^h$, gives $\alpha^l / \xi = \sum B^h m^h / m^l$. Using this

in the first-order conditions, tax rules are obtained:

$$\sum tS^h = \sum B^h m^h \left[\frac{x^l}{m^l} - \frac{x^h}{m^h} \right]. \quad (9)$$

These rules are of no direct use to us. Post-multiplication of (9) by t' , however, gives the quantity in which we are interested:

$$-T^l/m^l = \left[-\sum tS^h t' - \sum B^h T^h \right] / \sum B^h m^h. \quad (10)$$

This is the upper ceiling of the redistributive metric for the Ramsey–Rawls taxation scheme.

Conceptually, the taxes should be obtained as solutions to (9) and (1); and these, when substituted in (2), should yield the metric of redistribution. However, it is well recognized in the public finance literature that the nonlinearities in equations such as (9) make it impossible to obtain an analytical solution for taxes for the most general case. This difficulty leaves two options. The first option is to use a somewhat simplified structure of preferences for which an analytical solution is obtained. The second option is to make use of econometrically estimated parameters, from which tax rates are numerically computed. Both of these options will be utilized in the subsequent sections.

Before doing this, I shall provide a heuristic interpretation of the general expression for the upper ceiling of redistribution as it stands in (10). First, note that B^h , which is the net gain to h from unit income, corrected for marginal tax payment, is always positive. This is shown by differentiating the budget identity $x^h q' = m^h$, with respect to m^h . This gives $(\partial x^h / \partial m^h) q' = 1$. Substitution of $q' = p' + t'$ in this gives:

$$B^h = 1 - \frac{\partial x^h}{\partial m^h} t' = \frac{\partial x^h}{\partial m^h} p'.$$

Since $\partial x^h / \partial m^h > 0$ for normal goods, and $p' > 0$, it follows that

$$B^h > 0. \quad (11)$$

Therefore, the denominator on the right-hand side of (10) is positive, and it is the economy-wide aggregate income after correcting for the marginal tax payment. The first term in the bracket, $-\sum tS^h t'$, is twice the economy-wide Harbergerian deadweight loss [Harberger (1974, p.37)]. This is of second-order magnitude compared to the income, and is non-negative from the property of the Slutsky matrix. The second term in the bracket, $-\sum B^h T^h$, is

the negative of covariance between B^h and T^h , because $\sum T^h=0$ in (1). This term is likely to be positive because B^h , by its definition, is likely to decrease with an increase in T^h . These two economy-wide terms in the bracket in (10) are deflated to a kind of per unit income basis by the denominator.

A simpler expression is obtained for (10) if $B^h=B$ for all h . A sufficient condition for this to happen is that the slopes of the Engle curves for any good are identical across individuals. Using (1), (10) gives, $-T^l/m^l = -\sum tS^{ht}/B\sum m^h$. This is twice the Harbergerian deadweight loss per unit of income, divided by B . Now take an example. If as much as half of an increase in income is paid in increased taxes, i.e. $B=\frac{1}{2}$, and the Harbergerian loss is 20 percent of the income, then the maximum possible improvement in the real income of the worst-off is less than 80 percent. The redistribution, therefore, is unlikely to be large.

4. Two classes

Let us consider a simplification in which individuals are aggregated into two classes, rich and poor, with different preferences. The individuals are represented by $h=r$ and l , respectively, their population proportions are n^r and n^l . A further simplification is introduced here by aggregating commodities into two groups — necessities and luxuries. This merely requires the tax rates to be kept identical across commodities within a group, so that each group can be treated as a single Hicksian aggregate commodity. Needless to say, this model is simplistic. It should, however, be taken in the same spirit as many other models based on two-way classifications, e.g. two-sector models, which do manage to show the essential aspects of the problem.

The average consumption is given by $x_i = \sum n^h x_i^h$, and the average income is $m = \sum n^h m^h$. A ceiling on the metric is now obtained which is similar but sharper than (7). From (4), $(\theta_2 - \theta_1)/\theta_2 = 1/w_1$, as $w_2 = 1 - w_1$. Furthermore,

$$-T^l/m^l = -\sum_t \theta_t w_t^l = \theta_2 \{w_1^l [(\theta_2 - \theta_1)/\theta_2] - 1\}.$$

Substituting the term in square bracket from above, $-T^l/m^l = \theta_2(w_1^l/w_1 - 1)$. Now, if the rich are infinitely richer than the poor, i.e. $m^l/m^r \rightarrow 0$, then, $w_1 = w_1^r$. Hence,

$$-T^l/m^l = \theta_2[w_1^r/w_1 - 1]. \quad (12)$$

Take the first good as the necessity, i.e. $w_1^l > w_1^r$. It follows that $\theta_2 > 0$, if $-T^l/m^l > 0$. This is what we would expect: the welfare of the worst-off can be improved only if the luxury is taxed, and the necessity is subsidized. Using

$1 > \theta_2 > 0$ in (12)

$$-T^l/m^l < w_1^l/w_1^r - 1. \quad (13)$$

Note that this uppermost ceiling is narrower than the one obtained in (7). If the poor spend as much as 80 percent of their income on the necessity and the rich spend only 20 percent of their income on the necessity, then $-T^l/m^l < 3$. That is, the welfare of the poor can not be improved by more than three times. Furthermore, the same argument can be repeated if the income difference is finite, and if there are more than two classes. In this case, the ceiling is even smaller than the one obtained in (13).

As suggested above, more precise limits can be obtained by adding further structure to the problem. If the two groups have Cobb–Douglas demand functions, then $x_k^h = m^h \beta_k^h / q_k$, $\sum_k \beta_k^h = 1$, and $\beta_k^h > 0$. Let $m^r = \eta m^l$, and $n^r = \delta n^l$. For this case, analytical solutions for the tax rates are obtained from (1) and (9). These rates, when substituted into (2), give a direct expression for the metric in terms of the parameters:

$$\begin{aligned} I^l/m^l &= [\beta_1^l (1 + 1/\delta\eta) (\beta_1^r + \beta_1^l/\delta\eta)^{-1}]^{\beta_1^l} \\ &\times [\beta_2^l (1 + 1/\delta\eta) (\beta_2^r + \beta_2^l/\delta\eta)^{-1}]^{\beta_2^l} - 1. \end{aligned} \quad (14)$$

Take $\delta = 1$, $\eta = 20$, $\beta_1^l = 0.8$ and $\beta_1^r = 0.2$, that is, the population shares are equal, the rich are 20 times richer than the poor, and the budget shares are the same as in the last example. Then (14) yields, $I^l/m^l = 1.08$, which is obviously quite small. With the same set of parameters, but with $\eta \rightarrow \infty$, the metric $I^l/m^l = 1.3$. Thus, the redistribution remains meager even when income difference is infinite.

To ascertain that the above results are not due to the elasticity of substitution, σ , being one for the Cobb–Douglas case, I make use of the CES demand functions. The metric is calculated numerically when σ is perturbed around 1. From Pollak (1971)

$$x_i^h = m^h [(\beta_i^h)^{\sigma^h} q_i^{-\sigma^h}] \left[\sum_k (\beta_k^h)^{\sigma^h} q_k^{1-\sigma^h} \right]^{-1}; \quad \sum_k \beta_k^h = 1, \sigma^h > 0, \sigma^h < \infty. \quad (15)$$

If $p = [1, 1]$, $\beta_1^l = 0.8$, $\beta_1^r = 0.2$, $\delta = 1$, and $\eta = 20$, then $I^l/m^l = 0.78$, for $\sigma^l = \sigma^r = 0.75$. For $\sigma^l = \sigma^r = 1.15$, $I^l/m^l = 1.18$. Note that the metric is quite close to the corresponding Cobb–Douglas case.

The ceiling on the metric can also be obtained at the extremes of substitution possibilities, i.e. $\sigma = 0$ and ∞ . Only an outline of the arguments is presented here; the details can be filled in. For obtaining the uppermost

ceiling, I keep infinite income difference, i.e. $\eta \rightarrow \infty$. If $\sigma \rightarrow 0$, then the direct utility function is Leontieff, i.e. $U(x^h) = \min_k \{x_k^h / \beta_k^h\}$, and $x_i^h = m^h \beta_i^h / \sum_k \beta_k^h q_k$. Using this in (1), t_1 is first obtained as a function of t_2 . This relation is substituted into the indirect utility function, $v^l = m^l / \sum_k \beta_k^l q_k$, and v^l is maximized over t_2 , with an explicit constraint that $q_i \geq 0$. It is found that the post-tax price of the necessity becomes zero. Using this and (1), t_1 and t_2 are obtained, which are then used directly to obtain $-T^l/m^l$. For, $p = [1, 1]$, $\beta_1^l = 0.8$, and $\beta_1^r = 0.2$, this yields, $-T^l/m^l = 3$.

If $\sigma \rightarrow \infty$, then the direct utility function is linear, i.e. $U^h(x^h) = \sum_k \beta_k^h x_k^h$. This case is of special interest because a budget share may become zero due to the corner solutions. As $\eta \rightarrow \infty$, (1) becomes, $\sum_i \theta_i w_i^r = 0$. If $w_i^r = 1$ for either $i = 1$ or 2 , then $\theta_i = 0$, $t_i = 0$, and $T^r = 0$. This implies $T^l = 0$, from (1). I assume throughout that the post-tax prices are finite and positive. The welfare of l , therefore, can be improved only if the rich consume both goods. To make them do so, it is necessary to adjust taxes until $\beta_1^r / \beta_2^r = q_1 / q_2$. But this is not sufficient because the budget share becomes arbitrary in this case. Even if it is explicitly assumed that $w_i^r > 0$ for both goods, it is not guaranteed that $-T^l/m^l > 0$, because the tax rates are being set independently of l , as stated above. Recalling (12), this happens if $\beta_1^l > \beta_1^r$, which gives $w_1^l = 1$, and if $\theta_2 < 0$. Thus, if it is at all possible to improve the welfare of l , then the ceiling of the metric is given by (13) with the qualifications stated above.

Finally, the following result indicates that the above magnitudes remain unchanged even if the number of commodity groups is increased. For three commodity groups, the redistribution metric is obtained numerically for the Cobb–Douglas case, for $\beta_1^l = 0.6$, $\beta_2^l = 0.3$, and $\beta_3^l = 0.1$, and $\beta_1^r = 0.1$, $\beta_2^r = 0.3$, and $\beta_3^r = 0.6$. For $\delta = 1$ and $\eta = 20$ this yields, $I^l/m^l = 1.16$. The metric I^l/m^l is 1.45 when $\eta \rightarrow \infty$. For all of the cases above, which represent a wide class of specifications, we thus conclude that the redistribution possibility is quite limited.

5. Uniform preferences

The limit of redistribution is studied in this section by reverting back to an arbitrary number of classes of individuals while making a different simplification that all individuals have identical preferences and their Engle curves are linear. The corresponding expenditure function, called the Gorman form, is $E^h(q, v^h) = a(q) + v^h b(q)$, where $a(q)$ and $b(q)$ are concave and homogeneous of degree one in q . There are two justifications for using this form. First, most of the empirical work is based on the assumption of linearity of Engle curves; and ultimately the policy outcome of tax theory must depend on the available empirical work. Second, the above form is a second-order approximation to any arbitrary expenditure function [Deaton and Muellbauer (1980, pp. 144–145)]. The corresponding demand functions

are given by $x_i^h = a_i + (m^h - a)b_i/b$, where a_i and b_i are the respective derivatives of a and b with respect to q_i . The dependence of a_i , b_i , a , and b on the post-tax price q has been suppressed for notational convenience. a is figuratively called the 'subsistence expenditure', i.e. the expenditure at which the expenditure function yields zero utility. a_i is the of demand of good i at this expenditure, and may be called 'subsistence quantity'. In general, a and a_i 's can take any values.

From the public budget constraint (1), and the above demand functions, $\sum_i a_i t_i + C(m - a) = 0$, where $C = \sum_i b_i t_i / b$, and m is the average income. This gives, $\sum_i a_i t_i = -C(m - a)$. Similarly

$$-T^l = -\sum_i a_i t_i - C(m^l - a). \quad (16)$$

Substitution of the last expression for $\sum_i a_i t_i$ in (16) gives $-T^l = C(m - m^l)$. As $m > m^l$, the case of $C \leq 0$ must be ignored, otherwise $-T^l \leq 0$. Furthermore, it was earlier established in (11) that $B > 0$. By definition, $B = 1 - C$, which gives $1 > C$. Therefore, $1 > C > 0$.

Now, from the homogeneity of a in q ,

$$a = \sum_i a_i q_i = \sum_i a_i p_i + \sum_i a_i t_i.$$

Use of this in (16) yields

$$-T^l = (1 - C) \left(\sum_i a_i p_i - a \right) + C \left(\sum_i a_i p_i - m^l \right). \quad (17)$$

This has an interesting interpretation. The ceiling on the real income gain is a weighted average, with positive weights adding to one, of two expressions, and these two do not directly contain the slope parameters. Therefore, one could heuristically say that the redistribution possibility is mainly dependent on the intercept terms of Engle curves, and not so much on the slope terms. Furthermore, from (17) it is clear that

$$-T^l/m^l < \left(\sum_i a_i p_i - a \right) / m^l, \quad \text{if } m^l > a \quad (18a)$$

$$< \sum_i a_i p_i / m^l - 1, \quad \text{if } m^l < a. \quad (18b)$$

Now, Pollak (1971, 1976) exhaustively examines all admissible specifications of the Gorman cost function arising out of additive utility function. From these, we first find that the intercept terms a_i do not depend

on prices, i.e. they are constant parameters for almost all possible cases.¹ Secondly, we find that the admissible demand functions are defined either for $m^l > a$, or for $m^l < a$. That is, for any admissible demand system, we need to consider only one of (18a) and (18b), not both. For example, expression (18b) holds, i.e. $m^l < a$, in the case in which the income consumption curve radiates downwards from the bliss point (a_1, a_2, \dots) . In this case the upper limit of the metric can be obtained directly from the pre-tax prices alone, using (18b). No calculation of taxes is needed.

The demand systems which must satisfy (18a) are more important in practice. They include the linear expenditure system (LES), and the translated-CES system. For the latter, the income-consumption paths are translations of the CES demand functions (15), just as the LES income-consumption paths are translations of the Cobb-Douglas case. These two cases are quite prominent in empirical demand analysis. For these, $m^l > a$, $m^l > \sum_i a_i p_i$. Now, if $a > 0$, then (18a) gives

$$-T^l/m^l < 1.$$

We have an important result: *if the preferences are represented by the LES or the translated-CES systems, and if the 'subsistence expenditure' is positive, then the real income of the worst-off can not be improved by more than twice. A sufficient, but not necessary, condition for this is that 'subsistence quantities' are positive. Note that this result is entirely independent of the income distribution, of the parameters of demand functions, and of the social aversion to inequality.*

I would finally bring out an important point about the approach in this paper. Apart from the exaggeration created by the Rawls welfare function, I have also exaggerated the metric by taking its upper bounds. The actual redistribution is much smaller than these limits. This is now shown using the parameters of an LES system for the U.K. estimated by Pollak and Wales (1978). Not all a_i 's are not positive in this case, but the 'subsistence expenditure', a , is positive in what follows. From the last result, the metric is less than one. What is shown now is that it is actually *much smaller* than one.

The average sample income (expenditure), 392.8s/week, is taken as the economy-wide average income. I^l/m^l is calculated numerically from (1), (2), and (9), when m^l takes different values. This includes the lowest sample income, 232s/week. The results are presented in table 1. If the poorest person has 38 percent of the average income, then his real income rises by 14 percent. Indirect taxation makes practically no difference if the income of the poorest person is 60 percent or more of the average income. It is clear that

¹The only exception is an empirically uninteresting case, the details of which can be found in Pollak (1971, p. 405). This is excluded from further consideration.

Table 1
Maximum possible redistribution for the U.K., based on LES
consumption.

Income of the poorest, m^l	150	200	232 ^a	300	392.8 ^b
Real income gain to the poorest as a proportion of income, l^l/m^l	0.134	0.056	0.032	0.008	0

^aLowest sample income.

^bAverage sample income, m .

other reasonable values of the lowest and the average income would not change the conclusions at all. The results in this section, once again, bring out the inadequacy of indirect taxation as a redistributive instrument.

6. Discussion

First, an issue in this exercise is the identification of the worst-off individual. If preferences are identical, then the lowest income individual is the worst-off in any price regime. If preferences are not identical, then the society needs to agree about this matter. In practice, comparison among groups is based on broad features such as average income, family composition, and geographical location. Within these, it does not require an interminable discourse to agree upon the worst-off. For example, the rural poor are worse-off than the urban poor in developing countries because the former are eager to become the latter. Given any such agreement, the present approach holds if the price changes are only such that the worst-off remain the worst-off.

Secondly, the results would change if there are goods that the poor do not consume but the rich do, because then these goods can be used as perfect screening devices. But in this case, the government acquires a *direct* tax tool. Furthermore, it is important to remember that such purchase behavior is mostly due to the discreteness in the purchase quantities and its impact on the income–price regime of an individual. The treatment of such exclusive commodities therefore must be based on an explicit incorporation of discreteness. This would be a substantial extension, given the present state-of-the-art in dealing with discreteness. A different, and much simpler, extension will be to add leisure–commodity trade-off to the present model. This, however, is unlikely to change the results. Finally, note that the results here do not require normalization of utility functions, as is the case generally in the literature on taxation.

7. Conclusion

The objective of this paper has been to assess the redistribution capability of indirect taxes. This question is of great policy relevance in the countries which are constrained to depend primarily on indirect taxes; and such is indeed the case in a large number of countries. Despite its importance, the question has not been posed thus far in the literature.

My strategy in this paper has been to assess the redistributive possibilities after exaggerating them, because the actual possibilities must be less than these. For example, the most redistributive taxation scheme is adopted by considering the welfare of the worst-off individual only. The increase in the real income of the worst-off individual is taken as the metric of redistribution while the public budget is balanced. General expressions for the limits on this metric are obtained and interpreted. In addition, the ceilings of this metric are obtained explicitly for many specific models of the economy. Though these models are simplified versions of reality, they do provide clear answers to the question of our interest. It is found that the upper limits of redistribution is quite meager even though very large income differences might exist in the economy. As this is the outcome with the most redistributive taxation, it follows that the actual redistribution achieved through any real-world indirect taxation will be even smaller. It is therefore concluded that anyone seriously interested in redistribution has reasons to be quite skeptical of indirect taxes.

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