23

Can Government Collect Resources without Hurting Investors? Taxation of Returns from Assets

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This chapter shows that it may be possible to increase the government's resources, using taxes (and subsidies) on the returns from financial assets, without hurting investors. We work with a simple yet realistic set of taxes. The return from an asset is taxed at a constant rate. Different assets can be taxed at different rates.[1] All investors face an identical set of tax rates. It is not assumed that the government can tax every asset.

This analysis is conducted with heterogeneous investors whose investment opportunity set consists of one riskless and many risky assets. Here is a simple example of the results presented later. Suppose that: (1) investors are homogeneous, (2) there are no preexisting taxes, (3) there is one riskless and one risky asset, (4) the expected return from the risky asset exceeds the positive riskless return, and (5) investors have a nonzero portfolio weight on the riskless asset. Then, a small tax on the return from the risky asset, and a small subsidy on the riskless return, increases the government's resources without hurting investors. When we say that investors are not hurt, we mean that the changes in the tax rates under consideration do not reduce the level of the expected utility for any of the investors.

Several qualitative results are presented in more general settings, with many risky assets and with heterogeneous investors. A typical result is in the form of a single condition that is intuitive and informationally parsimonious. If this condition is satisfied by even one pair of assets, then a small change in the tax rates will increase the government's resources without hurting investors. Using numerical simulations, we also describe some large changes in the tax rates that yield the same. What we mean by small changes in the tax rates is that the tax rates are perturbed only in the local neighborhood of the pre-change tax regime, whereas no such restriction applies to large

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changes in the tax rates. We work with a small open economy that trades freely in financial assets with the rest of the world. We show that our results hold whether the asset markets are complete or incomplete. Later in this section, we summarize the reasons why the government may be able to increase its resources. To our knowledge, the results presented here are unavailable in the literature.

One of the concerns of the literature originated by Mossin (1968), Sandmo (1977), and Stiglitz (1969, 1972) has been to study the effects of taxes on an investor's portfolio weights (hereafter referred to as weights) on different assets. The focus here is on the effects of taxes on the welfare levels of investors and is, in this sense, orthogonal to the concern just noted. In a contribution to the analysis of taxation of risky assets, Auerbach (1981, 1) notes but does not analyze or establish the possibility that the government can raise resources without hurting individuals.

Since the taxes are imposed here on the returns from financial assets, the government's revenue is random. There are three main approaches adopted in the literature toward the treatment of random government revenue. The first is to ignore the revenue (see the review by Sandmo 1985 for such examples). This approach, though appropriate for analyzing many questions, is not applicable to the issues examined here. The second approach is to satisfy the government's budget constraint in each state-of-the-world, after reimbursing the government revenue to individuals on a lump-sum basis (see, e.g., Konrad 1991). While such analyses yield some economic insights, this characterization of the government may not be the most appropriate in many contexts. Among the reasons for this are: (1) lump-sum transfers are rarely feasible in practice; (2) governments typically have the ability to save and dissave, and to lend and borrow (including from nondomestic sources), across time and the states-of-the-world; (3) governments typically do not appear to balance their budgets on a state-by-state basis; and (4) the provision of public goods is a central activity of governments.

The third approach to the treatment of the government revenue is as follows. For brevity, let m denote the random government revenue. Then, an example of this approach is to specify that the government's resources are represented by the expected value of the revenue, $E[m]$, where $E[\ ]$ is the expectation operator (see Christiansen 1993, and Koskela and Kanniainen 1984). A motivation underlying this reduced-form approach is that the government can average out its random revenues across time and the states-of-the-world through saving, dissaving, borrowing, and lending, and that these averaging activities do not entail any net cost.

A more general version of this approach is to employ a transformation function on random government revenue and calculate the expected value of this function across the states-of-the-world (see Allingham 1972 and Stiglitz 1972 for similar formulations). We use this approach and represent the government's resources as $E[g(m)]$, where $g(m)$ is the transformation function. Our conclusions do not depend on virtually any property of the transformation function $g(m)$. Also, for our analysis, it is not necessary to specify how the government's resources are employed (for example, on which public goods and services, or on which other activities), provided that an increase in the government's resources, in itself, does not decrease the welfare of any investor.[2]

Can Private Entities Collect Resources?

The government's power of taxation plays an important role in our analysis. This is because, unlike the government, a private entity (such as an investment bank) in a competitive market may not be able to collect resources by introducing private taxes or subsidies. Consider this simple illustration. Suppose that an investment bank is willing to buy or sell the riskless asset at a return higher than the riskless rate prevailing in the market. Then, investors will buy the riskless asset from this bank and sell the corresponding quantity of this asset to a competing investment bank that is trading this asset at the riskless rate prevailing in the market. In this process, the former investment bank will incur a loss, the investors will make a profit, and thus this activity will not arise. Analogous reasoning applies to the situations in which an investment bank wishes to impose a private tax on the riskless asset, or a private tax or subsidy on a risky asset, or a combination of these.

Where Do the Government's Resources Come From?

Suppose that there are no preexisting taxes on any asset, and that the government introduces a tax on the return from one of the assets. By doing so, the government creates a new post-tax joint distribution of asset returns (referred to, in the rest of this subsection, as post-tax distribution). What matters to investors is this post-tax distribution. By altering the tax rates on the returns from various assets, the government can create many different post-tax distributions. Each such post-tax distribution will in general entail a different set of random payoffs to the government and investors. Therefore, given the characteristics of investors, including their respective preferences, and given the government's transformation function, some of these post-tax distributions may be such that each investor is as well-off in the post-tax regime as he is in the pretax regime, while the quantity of the government's resources in the post-tax regime is larger than that in the pretax regime. An analogous logic applies if there are preexisting taxes.

As noted earlier, the economy considered here is open and small. This polar abstraction is more reasonable for some countries, but obviously not for all, than another commonly used polar abstraction in which the economy is closed. Briefly discussed here are some of the implications of the abstractions that we have chosen to work with. An implication of the openness of the economy is that, in some states-of-the-world, it receives net resource inflows from the rest of the world, whereas the opposite happens in some other states-of-the-world. Another implication of openness is that the welfare of some of those in the rest of the world may be affected by a change in the home country's tax regime. Since we have not examined these welfare effects, we do not address the issue of a "free lunch." Given the openness of the economy, an implication of its smallness is that the pretax distribution of asset returns faced by the home country's investors is not affected by the change in the tax regime in the home country. A question that remains is whether some of the insights presented here have counterparts if the assumption of smallness is modified.

It should thus be apparent now that some of the critical elements of our framework (in particular, the openness and the smallness of the economy, and the specification of the government's resources) are fundamentally different from those that are typically present in the models that study the Pareto optimality (or its versions that are constrained in one manner or another) of a competitive equilibrium in closed economies (with or without complete asset markets). For this reason, our analysis and conclusions are not directly comparable to, and thus are not inconsistent with, those associated with the models just noted.

Organization of the Chapter

Section 1 begins the analysis with homogeneous investors. Section 2 presents the analysis with heterogeneous investors. In these two sections, the analysis deals with small changes in the tax rates. Section 3 summarizes some of our analysis of large changes in the tax rates. Section 4 presents some extensions of the analysis in sections 1 and 2. Section 5 concludes.

1 A Basic Model

We begin by considering homogeneous investors. Investors are risk-averse. The investment opportunities consist of one riskless and n risky assets. Depending on the context, an asset may be interpreted as an asset class rather than as an individual asset. None of the $n+1$ assets is redundant. The riskless asset is denoted by $i=0$, and it yields a pretax return x_0. Throughout the chapter, "return" means the net return, and a distinction is made, wherever it is necessary, between the pretax and the post-tax return from an asset. We assume that $x_0 > 0$; that is, the riskless return is positive. Other values of x_0 are considered later. The risky assets are denoted by $i = 1$ to n, and their pretax random returns are represented by the column vector $\tilde{x} \equiv (\tilde{x}_1, \ldots, \tilde{x}_n)'$. Throughout the chapter, a prime denotes the transpose of a vector. The expected values of the pretax returns from the risky assets are denoted by the column vector $x \equiv (x_1, \ldots, x_n)'$, where $x \equiv E[\tilde{x}]$. The investor's initial wealth is denoted by y, and $y > 0$. The investor's weight on asset i is a_i, where

$$a_0 = 1 - \sum_{k=1}^{n} a_k. \tag{1}$$

Taxes

The column vector $t \equiv (t_0, t_1, \ldots, t_n)'$ represents the tax rates on the returns from different assets. For any asset, the sign of its post-tax return is the same as that of its pretax return; that is $t_i < 1$.

In the present context, it is not meaningful to view a positive value of t_i as a "tax" on the i-th asset, or a negative value of t_i as a "subsidy." The effect of the sign of t_i depends on whether the investor is long or short on asset i. For example, if the investor is short on the riskless asset, then a positive value of t_0 reduces the cost of this borrowing, implying a subsidy. Alternatively, if the investor is long on the riskless asset, then a negative value of t_0 raises the return from this lending, also implying a subsidy. Hence, unless explicitly needed, we use the phrase "tax" to refer to a positive, zero, or negative value of any element of t.

Utility Levels of Investors

With full tax-loss offset, the random terminal wealth of the investor, denoted by Y, is

$$Y \equiv y\left\{1 + \sum_{i=0}^{n}(1 - t_i)a_i\tilde{x}_i\right\}. \tag{2}$$

For brevity in summations over assets, we sometimes write x_0 as $\tilde{x}_0$, as is done in (2) above, while keeping in mind that $\tilde{x}_0$ is nonrandom.

The utility function of the investor is $u(Y)$, which is increasing and strictly concave in Y. Let $U(t)$ denote the maximized level of the investor's expected utility. That is,

$$U(t) \equiv \max_{(a_1,\ldots,a_n)} E[u(Y)]. \tag{3}$$

It is assumed throughout the essay that an investor's optimization problem, such as (3), has a unique interior solution. The first-order conditions for the optimality of (3), taking (1) and (2) into account, are

$$E[\tilde{x}_j u_Y] = (1 - t_0)x_0 E[u_Y]/(1 - t_j), \qquad \text{for } j = 1 \text{ to } n, \tag{4}$$

where $u_Y \equiv \partial u/\partial Y > 0$. Define $U_j \equiv dU/dt_j$ for $j = 0$ to n. Then,

$$U_j = -ya_j(1 - t_0)x_0 E[u_Y]/(1 - t_j), \qquad \text{for } j = 0 \text{ to } n. \tag{5}$$

Expression (5) is derived in the appendix. From (5), we obtain the following to be used later:

$$U_j/U_i = (1 - t_i)a_j/(1 - t_j)a_i, \qquad \text{for } i \text{ and } j = 0 \text{ to } n, \tag{6}$$

where it is assumed that $a_i \neq 0$.

Government's Resources

We assume that taxes are the only source of government revenue. Thus, for example, we abstract from issues related to government bonds. We also assume that the government does not directly buy or sell financial assets. This is consistent with the practice in many modern market economies, especially concerning corporate and other nongovernment securities. Among the reasons, not modeled here, is that such a participation will make the government a stakeholder in private entities, which may have adverse market consequences.

The random government revenue is HM, where H is the number of investors, and M is the revenue collected from each investor. It follows that

$$M \equiv y \sum_{i=0}^{n} t_i a_i \tilde{x}_i. \tag{7}$$

Define

$$G(t) \equiv E[g(HM)], \tag{8}$$

where the nonrandom scalar $G(t)$ denotes the government's resources. No assumption is made about the function $g(\ )$ in (8), except that $g_M(0) > 0$, where g_M denotes the derivative of g with respect to its argument. That is, if the government's revenue is zero, then a small increase in this revenue raises the government's resources. As will be seen later, even this innocuous assumption can be dropped. Further, any monotonically increasing function of $G(t)$ will leave the analysis unchanged. It is assumed that: (1) a larger amount of the government's resources, in itself, does not hurt any investor, (2) any benefit that an investor gets from the government's resources is additively separable from his utility (3), and (3) the preceding benefit has no effect on the investor's portfolio decisions.

Define $G_j \equiv dG/dt_j$ for $j = 0$ to n. From (7) and (8), we obtain

$$G_j = yHE[(a_j\tilde{x}_j + \xi_j)g_M], \quad \text{where } \xi_j \equiv \sum_{i=0}^{n} t_i \tilde{x}_i \frac{\partial a_i}{\partial t_j}, \quad \text{for } j = 0 \text{ to } n. \tag{9}$$

Small Changes in the Tax Rates

Consider a small change in the tax rates t_i and t_j that keeps the utility level of an investor unchanged. If t_j is increased by a small amount

δt_j, then the change in t_i must be

$$\delta t_i = -U_j \delta t_j / U_i. \tag{10}$$

The corresponding change in the government's resources, using (10), is $\delta G = G_i \delta t_i + G_j \delta t_j = (G_j - G_i U_j / U_i) \delta t_j$. Define a metric

$$\phi(i, j) \equiv G_j - G_i U_j / U_i. \tag{11}$$

The substitution of (6) into (11) yields

$$\phi(i, j) = G_j - G_i (1 - t_i) a_j / (1 - t_j) a_i, \qquad \text{for } i \text{ and } j = 0 \text{ to } n, \tag{12}$$

where the G_i's are given by (9). It then follows that

If $\phi(i, j)$ is positive, then the government's resources increase from a small increase in the tax on the return from asset j, and a change in the tax on the return from asset i that leaves investors unhurt. (13)

Expression (13) yields

PROPOSITION 1. Suppose that there is an arbitrary set of preexisting taxes. Consider any pair of assets i and j, such that investors have a nonzero weight on asset i. Then, the government's resources become larger from a small increase in the tax on the return from asset j, and a change in the tax on the return from asset i that keeps investors unhurt, if the right-hand side of (12) is positive.

A noticeable feature of this proposition is as follows. To increase the government's resources without hurting investors, it is sufficient that only one of the $n(n+1)/2$ possible pairs of assets satisfies the required condition. The same observation applies to all of the propositions presented later here, with the exception of proposition 3.

Changes in the Tax Rates in the Vicinity of the No-Tax Regime

While proposition 1 holds for any preexisting taxes, we now consider small changes in the tax rates in the vicinity of the "no-tax regime"; that is, the regime in which there are no preexisting taxes. As one would expect, the resulting conclusions are qualitatively more transparent than proposition 1.

Since all tax rates are zero, (9) implies that $\xi_i = 0$ for $i = 0$ to n. From (7), $M = 0$. Hence, $g_M(HM) = g_M(0)$ is non-random. Substitution of these into (9) yields

$$G_j = yHg_M(0)a_jx_j; \tag{14}$$

that is, G_j is proportional to a_jx_j. Combining (14) with (12), and recalling that $g_M(0) > 0$, we obtain

$$\phi(i, j) > 0 \qquad \text{if } a_j(x_j - x_i) > 0, \tag{15}$$

and the following result.

PROPOSITION 2. Suppose that there are no preexisting taxes. Consider any pair of assets i and j, such that investors have nonzero weights on both assets. Then, the government's resources become larger from a small increase in the tax on the return from asset j, and a change in the tax on the return from asset i that keeps investors unhurt, if: (1) investors have a positive (negative) weight on asset j, and (2) the expected return from asset j is larger (smaller) than that from asset i.

This proposition is highly parsimonious in its informational needs. To implement this proposition, the only information that the government needs is investors' current weights and the expected returns from various assets. The same informational parsimony holds for all of the propositions presented later that deal with tax changes around the no-tax regime.

Consider now a special case of proposition 2 in which there is only one risky asset; that is, $n = 1$. In this case, it is reasonable to assume that the expected return from the risky asset, x_1, is larger than the riskless return, x_0. This assumption ensures that investors' weight on the risky asset is positive; that is $a_1 > 0$ (see Ingersoll 1987, 68). Recall (15), and substitute into it: $i = 0$, $j = 1$, $x_1 > x_0$, and $a_1 > 0$. This yields $\phi(0, 1) > 0$.

From (13), the preceding inequality implies a small tax on the return from the risky asset (i.e., $\delta t_1 > 0$). In turn, a subsidy on the riskless return is required to keep investors unhurt. This is because, from (6) and (10), $\text{sgn}\{\delta t_0\} = -\text{sgn}\{a_0\}$, given that $a_1 > 0$. Thus, δt_0 will be negative or positive, depending on whether investors are long or short on the riskless asset. As was noted earlier, in either case, this implies a subsidy on the riskless return. These observations lead to the following conclusion.

PROPOSITION 3. Suppose that (1) there are no preexisting taxes, (2) the investment opportunities consist of one riskless and one risky asset, (3) the expected return from the risky asset exceeds the positive riskless return, and (4) investors have a nonzero weight on the riskless asset. Then, the government's resources become larger from a small tax on the return from the risky asset and a small subsidy on the riskless return that keeps investors unhurt.

Under the plausible conditions noted here, proposition 3 guarantees that the government's resources will increase, without hurting investors, if the investment opportunity set consists of a riskless and a risky asset. Some additional observations on this proposition are presented in Sah and Wada (2001).

Complete versus Incomplete Asset Markets

Recall the assertion made in the introduction that our results hold in complete as well as incomplete asset markets. To confirm this, note that there is no redundant asset in our analysis with one riskless and n risky assets. Also, we have placed no restriction on the number of the states-of-the-world. Thus, our analysis is for complete asset markets if there are only $n+1$ states-of-the-world. Alternatively, our analysis is for incomplete asset markets if the number of the states-of-the-world is larger than $n+1$.

2 Heterogeneous Investors

This section considers heterogeneous investors. Investors are identified by the superscript $h = 1, \ldots, H$. For brevity, the range of summation over investors is often suppressed. For investor h, the initial wealth is y^h (where $y^h > 0$), the random terminal wealth is Y^h, the utility function is $u^h(Y^h)$, the weight on asset i is a_i^h, and the maximized level of the expected utility is $U^h(t)$. Then, analogous to (1), (2), (3), and (6), we respectively have

$$a_0^h = 1 - \sum_{k=1}^{n} a_k^h, \tag{16}$$

$$Y^h \equiv y^h \left\{ 1 + \sum_{i=0}^{n} (1 - t_i) a_i^h \tilde{x}_i \right\}, \tag{17}$$

$$U^h(t) \equiv \max_{(a_1^h, \ldots, a_n^h)} : E[u^h(Y^h)], \qquad \text{and} \tag{18}$$

$$U_j^h/U_i^h = (1-t_i)a_j^h/(1-t_j)a_i^h, \qquad \text{for } i \text{ and } j = 0 \text{ to } n. \tag{19}$$

The government's resources are $G(t) \equiv E[g(\sum_r M^r)]$, where $M^h \equiv y^h \sum_{i=0}^n t_i a_i^h \tilde{x}_i$ is the revenue collected from investor h. It follows that

$$G_j = E\left[\sum_r y^r\{a_j^r\tilde{x}_j + \xi_j^r\}g_M\right],$$

$$\text{where } \xi_j^h \equiv \sum_{i=0}^n t_i\tilde{x}_i\frac{\partial a_i^h}{\partial t_j}, \text{ for } j = 0 \text{ to } n. \tag{20}$$

We work with the HARA-class utility functions, defined by the equation

$$-u_Y^h(Y^h)/u_{YY}^h(Y^h) = \varepsilon_1^h + \varepsilon_2 Y^h, \tag{21}$$

where $u_{YY}^h \equiv \partial^2 u^h/\partial(Y^h)^2$, and ε_1^h and ε_2 are parameters. Note that the heterogeneity in the investors' preferences is somewhat limited here because the parameter ε_2 is the same for all investors. We later present some comments on the HARA-class and note that our results also hold for some specifications other than that in (21).

Define

$$\alpha_k^h \equiv a_k^h/(1-a_0^h), \qquad \text{for } k = 1 \text{ to } n. \tag{22}$$

That is, α_k^h is the amount that investor h invests in the risky asset k as a fraction of his total wealth invested in all risky assets. For brevity, we refer to α_k^h as the "sub-weight" of investor h on the risky asset k. By definition, $\sum_{k=1}^n \alpha_k^h = 1$.

As is well known, (21) yields the two-fund separation property. Formally,

$$\alpha_k^h = \alpha_k, \qquad \text{for any } k = 1 \text{ to } n, \text{ and for all } h. \tag{23}$$

That is, the subweight on any risky asset k is the same for all investors (see Rubinstein 1974 at 227–228).

A property of the subweights α_k's is as follows. For even one α_k to be nonzero, as well as nontrivial (in the sense that this nonzero value of α_k has any impact on the terminal wealth), it is necessary that $a_0^h \neq 1$ for all h.[3] That is, an investor does not invest all of his wealth in the riskless asset. Accordingly, we assume that $a_0^h \neq 1$ for all h. Using this, (19), (22), and (23), we obtain

$$\phi^h(k,\ell) = G_\ell - G_k(1-t_k)\alpha_\ell/(1-t_\ell)\alpha_k, \qquad \text{for } k \text{ and } \ell = 1 \text{ to } n, \tag{24}$$

where the G_k's are given by (20). An important property of (24) is that the metric $\phi^h(k, \ell)$ is the same for all investors.

Next, as shown in the appendix, the evaluation of (24) in the no-tax regime yields

$$\phi^h(k, \ell) > 0 \text{ for all } h, \qquad \text{if } \left[\sum_r y^r(1 - a_0^r)\right] \alpha_\ell(x_\ell - x_k) > 0. \tag{25}$$

Note that $\sum_r y^r(1 - a_0^r)$ is the sum of all investors' initial wealth invested in all of the risky assets. We assume here that this sum is positive though, as shown later, an analogous result holds even if this sum were negative. Accordingly,

$$\phi^h(k, \ell) > 0 \qquad \text{if } \alpha_\ell(x_\ell - x_k) > 0. \tag{26}$$

We thus have the next two propositions.

PROPOSITION 4. Suppose that there is an arbitrary set of preexisting taxes, and that heterogeneous investors have the HARA-class utility functions defined by (21). Consider any pair of risky assets k and ℓ, such that investors have a nonzero weight on asset k. Then, the government's resources become larger from a small increase in the tax on the return from asset ℓ, and a change in the tax on the return from asset k that keeps each investor unhurt, if

the right-hand side of (24) is positive. (27)

A notable aspect of this proposition is as follows. Here, there is an arbitrary number of heterogeneous investors. Yet there is only one condition, namely (27), that needs to be satisfied (and not, say, one condition for each investor) for the government's resources to increase without hurting investors. Moreover, as noted earlier, it is sufficient that this one condition be satisfied for any one of the many possible pairs of assets. The same observation applies to the other propositions that deal with heterogeneous investors, namely, the next proposition and proposition 6 presented later.

PROPOSITION 5. Suppose that there are no preexisting taxes, and that heterogeneous investors have the HARA-class utility functions defined by (21). Consider any pair of risky assets k and ℓ, such that investors have nonzero weights on both assets. Then, the government's resources become larger from a small increase in the tax on the return from asset ℓ, and a change in the tax on the return from asset k that keeps each investor unhurt, if (1) investors have a positive (negative) subweight on asset ℓ, and (2) the expected return from asset ℓ is larger (smaller) than that from asset k.

Isoelastic Utility Functions

Propositions 4 and 5 apply to taxes on any pair of risky assets, but not to taxes on a pair consisting of the riskless and any one of the risky assets. We now present a result that includes pairs of taxes of the latter type. Suppose that the investors' utility functions are isoelastic; that is, $u^h(Y^h) = (Y^h)^{1-\varepsilon}/(1-\varepsilon)$, where $0 < \varepsilon < \infty$ is the parameter of relative risk-aversion. This is a special case of (21) with $\varepsilon_1^h = 0$, and $\varepsilon_2 = 1/\varepsilon$. This utility function, sometimes called the CRRA utility, includes the log utility function as a special case. Accordingly, propositions 4 and 5 hold in the present case. A further implication of the isoelastic utility function is that all investors have the same weight on any given asset. That is,

$$a_i^h = a_i, \qquad \text{for any } i = 0 \text{ to } n, \text{ and for all } h. \tag{28}$$

Define $y \equiv \sum_r y^r/H$ as the average initial wealth per investor. Then, using (19) and (28), we obtain

$$\phi^h(i, j) = G_j - G_i(1 - t_i)a_j/(1 - t_j)a_i, \qquad \text{for } i \text{ and } j = 0 \text{ to } n, \tag{29}$$

where the G_j's are given by (9). Thus, the right-hand side of (29) is the same as that of (12).

Further, in the no-tax regime, we have the same expression as (15), but now it applies to all investors. That is,

$$\phi^h(i, j) > 0 \qquad \text{if } a_j(x_j - x_i) > 0. \tag{30}$$

These observations lead to

PROPOSITION 6. Suppose that heterogeneous investors have isoelastic utility functions. Then, the conclusions of propositions 1, 2, and 3 hold without any modification.

The reason underlying proposition 6 is simple. To an investor, what matters for the present analysis is his set of portfolio weights. For investors with isoelastic utility functions (with heterogeneous levels of wealth, but the same parameter of relative risk-aversion), this set is identical across investors.

3 Large Changes in the Tax Rates: A Summary

In the preceding two sections, the focus was on small changes in the tax rates. Separately, we have analyzed whether some large changes in the tax rates can increase the government's resources. Due to space constraints, only a verbal summary of this analysis is presented below; the key details are in Sah and Wada (2001).

Consider the following framework. There are many risky assets and one riskless asset. Investors have mean-variance (M-V) utilities. That is, the welfare level of investor h is $E[Y^h] - v^h\, Var[Y^h]/2$, where his post-tax random terminal wealth Y^h is described by (17), and v^h is his risk-aversion parameter. Note that this M-V specification is different from the one that arises if a quadratic utility function (which is a special case of the HARA-class utility functions considered earlier) is used. Investors are heterogeneous in their initial wealth levels and in their risk-aversion parameters. The government's resources are defined in an M-V fashion; in particular, $G \equiv E[\sum_h M^h] - \rho\, Var[\sum_h M^h]/2$, where $M^h \equiv y^h \sum_{i=0}^{n} t_i a_i^h \tilde{x}_i$ is the random government revenue collected from investor h. For simplicity, we deal only with positive values of ρ.

We know from mean-variance portfolio analysis (Ingersoll 1987, 88) that there is an efficient-frontier that represents the feasible and relevant portfolio choices of all investors. A commonly used graphical description of this efficient-frontier is that the standard deviation of the portfolio return is on the horizontal axis, the resulting expected return of the portfolio is on the vertical axis, and the efficient-frontier is an upward sloping straight line of which the intercept is the riskless return (the relevant riskless return here is the post-tax one). The optimal choice of any one investor is a point on the efficient-frontier, and the optimal choices of a heterogeneous group of investors will in general be represented by many different points on the efficient-frontier. Therefore, the welfare level of each of the investors can be kept unchanged, while changing the tax vector, by considering those combinations of changes in the tax rates which leave the efficient-frontier unaltered.

Next, using the standard analytics of mean-variance portfolio analysis, it can be shown that, in general, a change in the tax rate on the riskless return (recall that t_0 represents this tax rate) affects both the intercept and the slope of the efficient-frontier, whereas a change in the tax rate on the return from a risky asset (recall that $(t_1, \ldots, t_n)$

represent these tax rates) affects the slope but not the intercept of the efficient-frontier. Therefore, in our analysis, we keep t_0 unchanged, while examining combinations of changes in $(t_1, \ldots, t_n)$ such that the efficient-frontier remains unaltered. Note that this analysis does not require any information on the initial wealth levels of investors, because this information is not relevant for constructing the efficient-frontier.

In our numerical simulations, the investment opportunity set consists of one riskless and two risky assets. The tax rate t_0 is set at zero, and the only changes considered are those in t_1 and t_2. To keep the efficient-frontier unaltered, only one of these two tax rates, t_1 and t_2, can be changed independently of the other. We define the prechange regime to be the one in which there are no taxes; i.e., $t_1 = t_2 = 0$. It follows that $G = 0$ in this benchmark regime. The data on annual real returns are taken from Ibbotson (1996), for each year from 1926 to 1995. These data for the United States are treated as pretax data. The riskless return corresponds to that from the T-bills. The returns from the two risky assets correspond respectively to those from (1) the T-bonds, and (2) an equally weighted portfolio of small- and large-cap stocks. We find a number of combinations of t_1 and t_2 for which is G positive. That is, the government's resources increase as a result of nonlocal changes in the tax rates on the returns from assets. This conclusion holds for several sets of values of the parameters ρ and $(\nu^1, \ldots, \nu^H)$.

4 Some Extensions

This section summarizes some extensions of the analysis presented in sections 1 and 2. Each extension is independent of the others.

Investor's Preferences

In a part of section 2 it was assumed that investors have the HARA-class utility functions defined by (21). This class includes many of the utility functions commonly used in financial economics, such as quadratic, cubic, exponential, and isoelastic. Further, what matters for the corresponding results in that section is the two-fund separation property and not the nature of the underlying utility functions. Thus, analogs of these results will hold under all those conditions in which investors exhibit the two-fund separation property. Ingersoll

(1987, 164) summarizes several such conditions in terms of restrictions on investors' preferences and/or the distribution of the asset returns.

Riskless Return

Our analysis has been based on the assumption that the riskless return is positive. For expositional brevity, we do not consider negative values of the riskless return, though such values do not alter the economic conclusions of the essay. If the riskless return is zero, then the analysis becomes considerably simpler. Note from (5) that if $x_0 = 0$, then $U_j = 0$, for $j = 0$ to n. That is, taxes have no effect on the expected utility of an investor (which was originally established by Sandmo 1977, 1989). Thus, the government's resources can be increased, whether or not there are preexisting taxes, if at least one of the G_j's is nonzero, for $j = 0$ to n. This is because a small increase or decrease in the tax rate t_j will increase G if G_j is respectively positive or negative. The implementation of this conclusion is especially easy for changes in the tax rates in the vicinity of the no-tax regime. For instance, with homogeneous investors, it follows from (9) that, in this case, $\text{sgn}\{G_j\} = \text{sgn}\{a_j x_j\}$.

Zero Weight on an Asset

For simplicity, consider homogeneous investors. It follows from (5) that if an investor has a zero weight on asset i, then his expected utility remains unchanged due to a change in the tax rate t_i. Therefore, using the reasoning presented in the preceding paragraph, the government's resources can be increased through a small increase or decrease in t_i, depending on whether G_i is positive or negative. This conclusion, however, does not translate for use around the no-tax regime because, in this case, (9) implies that G_i is zero.

Other Assumptions

We assumed earlier that $g_M(0) > 0$; that is, a slight increase in the government revenue increases the government's resources if the current government revenue is zero. Consider the opposite assumption; namely that $g_M(0) < 0$. This reverses the sign of the inequalities (15), (26), and (30). This reversal merely requires a rewording of the corresponding propositions that deal with tax changes around the no-tax regime.

An assumption used in deriving proposition 5 was that $\sum_r y^r(1 - a_0^r) > 0$; that is, the amount invested by all investors in all of the risky assets is positive. Suppose instead that the sign of the preceding expression is negative. Then, the sign of the inequality (26) will be reversed, resulting in a simple restatement of proposition 5.

5 Concluding Remarks

Ever since the work of Harberger, we have known that the garnering of resources for public goods and services imposes direct costs on private individuals, and, in addition, can entail significant indirect costs. If the level of public resources can be increased for a given set of private costs, then it is typically the case that the private costs can be reduced for a given level of public resources. While we recognize this, it is not our aim to look for free lunches. This chapter does not even raise the issue of a free lunch, because we have looked at changes in the tax regime in a small open economy without accounting for the effects that such changes may have on the rest of the world, and how the rest of the world might respond to such changes. At the same time, it is the case that a small open economy is a more appropriate abstraction for many countries than another commonly used abstraction, namely, that of a closed economy. It is also the case that, in many contexts, policy decisions are made within a country without much attention to its implications on the rest of the world and, for one reason or another, without significant responses from the rest of the world. The possibility of being able to reduce the private costs of raising public resources appears intriguing to us, even if it may be meaningful only within specific contexts such as the one just noted.

Appendix

DERIVATION OF (5). The use of the envelope theorem on (3), while using (2), yields

$$U_j = -ya_jE[\tilde{x}_j u_Y], \qquad \text{for } j = 0 \text{ to } n. \tag{A1}$$

Substitution of (4) into (A1) yields the desired expression.

DERIVATION OF (25). Here, each element of t is zero. From its definition, $M^h = 0$. Thus, $\sum_r M^r = 0$, and $g_M(0)$ is nonrandom. Further, $\xi_j^h = 0$ from (20). Substitution of these and (22) and (23) into (20) yields: $G_j = g_M(0)[\sum_r y^r(1 - a_0^r)]x_j\alpha_j$. This and (24) yield $\phi^h(k, \ell) > 0$ for all h, if $g_M(0)[\sum_r y^r(1 - a_0^r)]\alpha_\ell(x_\ell - x_k) > 0$. The desired expression follows since $g_M(0) > 0$.

Acknowledgments

We present this essay to celebrate and honor Joe Stiglitz. From Raaj, it is also a token of appreciation to a friend. Joe's intense scholarly originality has deeply impacted many parts of economics. As a public servant and social thinker, he has unflinchingly created alternative visions for a better world. More than anything else, it was Joe's idealism that led him to become an economist. It is heartwarming that, unlike many whose idealism withers with the passage of time, Joe's has flourished richly. And his ever-present warmth and liveliness. God bless!

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Notes

1. Gains from different asset classes are taxed at different rates in a number of countries. Price Waterhouse (1996) provides many examples.

2. Note that the government's instruments in the present analysis are quite limited. For example, the tax rates on the returns are proportional. Our conclusions will only be reinforced if the set of instruments that the government can use were larger (e.g., nonlinear tax rates on the returns from assets) than that specified here. We work with the simplifying assumption that the investors pay taxes only to their home government, and that no foreigners pay taxes to this government, but these aspects can easily be modified. We also assume that the government can prevent the domestic investors' tax noncompliance (including through arbitrage with the rest of the world); for example, by requiring that these investors are served only by domestically regulated brokers.

3. This can be seen by using (16) and (22) to rewrite (17) as $Y^h = y^h\{1 + (1 - t_0)a_0^h x_0 + (1 - a_0^h)\sum_{k=1}^{n}(1 - t_k)\alpha_k \bar{x}_k\}$. Suppose $\alpha_\ell \neq 0$. Then, the value of α_ℓ has no impact on Y^h if $a_0^h = 1$.

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