Class struggle: Some diluting effects of inter-generational mobility

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University of Chicago, Harris School of Public Policy. Working paper series 03.01. November 2002.

ABSTRACT

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Our neoclassical analysis posits that the welfare of one's own progeny matters to each individual. Taking this into account, each of the classes (namely, the poor and the rich) chooses how much resources to spend on influencing the stochastic outcomes of class struggle in their respective favors. These conflicts are depicted as non-cooperative games.

An implication of the inter-generational concerns is that a society's inter-class economic mobility across generations turns out to be a central determinant of the classes' choices. Among the results that we present is that, in a society with greater class mobility: (i) the poor spend less on class struggle, (ii) the rich may spend less or more, but (iii) the probability of a successful class struggle is lower. Our analysis also suggests that class struggle might disappear altogether in societies with high levels of class mobility.

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Revised: November 2002

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*We thank Gary Becker, Allison Garrett, Casey Mulligan, and Jian Sun for their valuable suggestions. The authors are solely responsible for the contents of this paper

I. Introduction

Marx seems to have believed that his most original idea, in his large corpus of writings, was that of class struggle (Wilson (1972, p. 184)). According to him, all societal history was, and will in the future be, merely be a succession of struggles between classes.[1] Many authors have elaborated upon such themes within Marxist frameworks; some have used these as prisms to interpret various past events, especially outside North America.

Becker (1983, 1985) developed an economic approach to analyze competition among societal groups, motivated by each group's desire to seek influence and benefit. We adopt this approach to portray class struggle as a conflict between the poor and the rich. Both groups spend optimal amounts of resources to influence the stochastic outcomes of class struggle in their respective favors. Like Becker, we abstract from intermediaries such as unions, political parties, voting-based legislatures, and other coalition-forming mechanisms. Such intermediaries appear to have some effect of ameliorating potential antagonisms between the poor and the rich, especially in advanced democracies such as those in North America and Northern Europe. On the other hand, in many countries, the primary role of these intermediaries appears to be to pursue the objectives of one or the other class. Hence, after factoring out any independent role that such intermediaries might have, there remains a potential for group-to-group antagonism between the poor and the rich. The present paper focuses on this part.

A cornerstone of our neoclassical analysis consists of the inter-generational con-

[1] An annotated selection of Marx's writings on class struggle and related topics is Elster (1986, pp. 223-256). A modern commentary on these writings is Elster (1985, pp. 318-397).

cerns. Beginning with Barro (1974) and Becker (1974), it is now well established that individuals care about their respective progenies. Moreover, the analyses of a wide variety of otherwise unrelated economic questions have turned out to be richer and more fruitful when inter-generational concerns are incorporated.

>From the point of view of an individual in a given society, a key part of the inter-generational picture is the society's level of inter-class economic mobility across generations (for brevity, we hereafter refer to it as "class mobility"). For example, what does a poor person perceive to be the probability that his progeny will become rich? It is important to note here that the class mobility of any given individual is influenced by that individual's intrinsic characteristics, and by the various dimensions of his human capital. However, inter-class rigidities may also affect the access of individuals to the sources of acquiring human capital. For example, such has been the case for centuries in parts of South Asia where there is pervasive caste system.

An example of why class mobility plays an important role in our analysis is as follows. In the first instance, a greater class mobility is likely to induce the poor to spend fewer resources on class struggle, which will have the tendency to lower the probability of a successful class struggle.[2] On the other hand, in the first instance, a greater class mobility is likely to also induce the rich to spend fewer resources on class struggle, but this will have the tendency to raise the probability of a successful class struggle. A part of this paper analyzes these forces and, equally importantly, the interaction effects between the two classes. These interactions are depicted as non-cooperative games.

[2] We use the word "successful" only for brevity, in the inert sense that, under this outcome, the poor believe that they have achieved their perceived goals, and not in the sense of a society becoming better along one or another dimension.

Among the results that we present are that, in societies with greater class mobility: (i) the poor spend less resources on class struggle, (ii) the rich may spend less or more, but (iii) the probability of a successful class struggle is lower. Our analysis also suggests that class struggle might disappear altogether in societies with high levels of class mobility. The intuitions underlying these and other results are described in the paper. To our knowledge, the role of inter-generational mobility in the context of class struggle has not been explicitly studied in the literature.[3]

Anecdotal writings suggest that the U.S. had a significantly higher level of class mobility in the 19th century compared to Western Europe (Tocqueville (1849)). Mulligan (1997) has studied intergenerational mobility in the U.S. using direct micro measures of economic status such as consumption, earnings, income and wealth. He also compares studies by other authors, including on other countries, that use different kinds of methods and data. His study points to a dramatic decline in wealth inequality in the U.S. and the U.K. since the mid-19th century, which is consistent with increased class mobility during this period. He also concludes that, in more recent times, there is no noticeable difference in mobility between the U.S. and a

[3] On the other hand, the present paper's emphasis on intergenerational motives and class mobility is consistent with, or complementary to, some of the existing analytical approaches towards understanding class conflict. For instance, a concern of Przeworski and Wallerstein (1982) is to delineate some of the circumstances under which compromises between two classes might or might not be feasible. Roemer (1995) analyzes why the poor in democracies do not expropriate the rich through electoral and legislative processes. A part of his argument is that the right-wing parties are fiscal and moral (or religious) conservatives, and the left-wing parties are fiscal and moral liberals, while many poor may be fiscal liberals but moral conservatives. In a separate analysis, Roemer (1985) studies, among other things, the nature of the agenda that a leader will propose to the poor to form a revolutionary coalition, and the strategy that his opponent will follow.

sample of Western European countries. On the other hand, comprehensive studies of mobility, extending over significant lengths of time, are typically unavailable for poor agricultural countries. It is possible that some of these countries have lower class mobility than modern industrialized countries.

Many episodes of conflicts between the poor and the rich have been noted in the social history of the last few centuries (see, for instance, Hobsbawm (1963) and Thompson (1968)). Some fragmentary evidence exists for more ancient periods (see, for example, de Ste. Croix (1981)). Such struggles can continue at a subterranean level, or can sometimes take the form of open conflicts. We view these differences as different manifestations of the same underlying phenomenon. This notwithstanding, such conflicts have historically been probably more intense in Western European countries than in North America. Likewise, such conflicts have probably been, and in some cases may presently be, more intense in poor agricultural countries than in rich industrial countries. For example, this century's two major episodes of open class conflicts (the October Revolution in Russia and the Maoist Revolution in China) occurred in countries that were contemporaneously poor.

The paper is organized as follows. Section II presents and analyzes a basic model. Section III extends the preceding analysis in several ways. Section IV presents concluding remarks, including some on possible venues for future research.

II. A Basic Model

The economy consists of two classes, poor and rich, denoted respectively by the superscript $h = P$ and R, with populations N^P and N^R. Within a class, all members are homogenous. For now, each of the two classes have the same number of members;

that is $N^P = N^R$. An individual lives for one generation, and is then followed by one progeny of his in the next generation. For simplicity, we presently consider only two generations; a more general model is described later in the paper.

Based on his benefits and the probability of a successful outcome, a poor person in the first generation decides on the resources (e.g., time, effort, emotional energy) that he will spend on class conflict. A rich person in the first generation does the same, except that he wishes to reduce the probability of a successful outcome. The probability of a successful outcome is jointly determined by the two sets of resources just mentioned. Neither the poor nor the rich undertake free-riding in determining the resources that they will spend on class struggle. Free-riding is considered later in the paper.

If the class struggle has a successful outcome, then no direct gains or losses accrue from this outcome to the first generation. On the other hand, the next generation faces a changed society, with an altered level of total societal resources, in which a poor person has the same income as a rich person (all incomes in this paper are full incomes). This equality of the incomes of a poor and a rich person is a simplifying assumption. The content of our analysis remains unaffected if, instead, it were assumed that a successful class struggle leads to a significant reduction in the difference between the incomes of a poor and a rich person.

If the class conflict has an unsuccessful outcome, then the status-quo is maintained. That is, a poor and a rich person in the next generation have respectively the same incomes as a poor and rich person in the first generation. However, whether a poor or rich person's progeny turns out to be poor or rich in the next generation is stochastically determined by the class mobility of the society.

Let m denote the probability that, under the status-quo, a poor person's progeny becomes rich, and a rich person's progeny becomes poor.[4] A natural restriction on m is that $0 < m < 1/2$. The reason for m to be positive is that it is difficult to imagine a society in which there is no class mobility at all. At the other extreme, if $m = 1/2$, then half of the progeny of those currently poor will end up being rich, and vice versa. This perhaps exaggerates the reality even for countries that have high levels of social mobility.

The exogenous income of a poor person is w^P, and that of a rich person is w^R, where $w^P < w^R$. In the first generation, a poor person spends c^P on class struggle, where $0 \leq c^P < w^P$, and a rich person spends c^R, where $0 \leq c^R < w^R$. The probability that the class struggle will have a successful outcome is $Q(N^P c^P, N^R c^R)$. Since both groups have the same populations, $c \equiv c^P + c^R$ is an appropriate indicator of the total societal resources spent on class struggle.

If the class struggle is successful, then each member of the next generation, whether a progeny of rich or poor, has income w such that $w^P < w < w^R$. For now, it need not concern us as to how the value of w is determined; this will partly depend on how the total societal income is altered by a successful class struggle.

If the class struggle is not successful, then the status-quo prevails. With probability m, the progeny of a poor becomes rich, and the progeny of a rich becomes poor. We presently assume that all individuals are risk-neutral; the role of risk-aversion is discussed later.

Let y^P and y^R respectively denote the utilities of a first-generation poor and rich

[4] The substance of the present analysis does not change if the probability of a poor person's progeny becoming rich is different from the probability of vice versa.

if the class struggle is successful. Let $0 < \delta < 1$ denote the discount factor that a first-generation individual applies to the expected income of his progeny. Then,

$$y^h \equiv (w^h - c^h) + \delta w, \text{ for } h = P \text{ and } R. \tag{1}$$

Let Y^P and Y^R respectively denote the utilities of a first-generation poor and rich if the class struggle is not successful. Then,

$$Y^h \equiv (w^h - c^h) + \delta \left[(1-m)w^h + mw^\ell\right], \tag{2}$$
$$\text{for } h = P \text{ and } R,\ \ell = P \text{ and } R, \text{ and } \ell \neq h.$$

Using (1) and (2), it follows that the expected utility of a poor person and a rich person in the first generation are given by

$$v^h(c^P, c^R) \equiv Y^h + Q(N^P c^P, N^R c^R) g^h, \text{ for } h = P \text{ and } R, \tag{3}$$

where $g^h \equiv y^h - Y^h$. Note that g^h denotes the gain to a first-generation person in class h from a successful class struggle compared to an unsuccessful one. We assume that the value of a successful class struggle is positive for a poor person, and negative for a rich person. That is, $g^P > 0$ and $g^R < 0$. If this were not so, then neither the poor nor the rich would spend resources on class struggle. Accordingly, from (1) and (2),

$$g^P = \delta[w - \{(1-m)w^P + mw^R\}] > 0, \text{ and}$$
$$g^R = \delta[w - \{mw^P + (1-m)w^R\}] < 0. \tag{4}$$

For later use, we define some additional variables. Since both groups have equal populations, we use $g \equiv g^P + g^R$ as the indicator of the societal gain from a successful class struggle, as compared to an unsuccessful one. We assume that $g < 0$; that is, a successful class struggle lowers total societal income. The Soviet and the Chinese experiences largely support this assumption.

Next, define $g^h_m \equiv \partial g^h / \partial m$, for $h = P$ and R, which are the partial derivatives of the gain from a successful class struggle with respect to the level of class mobility. Also, define $g_m \equiv \partial g / \partial m$. From (4),

$$g^P_m = \delta(w^P - w^R) < 0, \text{ and } g^R_m = \delta(w^R - w^P) > 0. \tag{5}$$

It follows from (5) that

$$g^P_m = -g^R_m, \text{ and so } g_m = 0. \tag{6}$$

Expression (5) has an intuitive meaning. A successful class struggle has a lower value to a poor person in a more mobile society. This is because a larger mobility raises the probability that his progeny will be rich, thereby increasing his utility in the case of status-quo, while keeping unchanged his utility in the case of a successful class struggle. For the opposite reason, a successful class struggle has a higher value to a rich person in a more mobile society. A larger mobility raises the probability that a rich person's progeny will be poor, thus lowering his utility in the case of status-quo, while leaving unchanged his utility in the case of a successful class struggle.

The probability of a successful class struggle, $Q(\cdot, \cdot)$, is assumed to be increasing (at a decreasing rate) in the spending by the poor, and decreasing (at a decreasing rate) in the spending by the rich. Define $Q_P \equiv \partial Q / \partial (N^P c^P)$, $Q_R \equiv \partial Q / \partial (N^R c^R)$, and $Q_{PP} \equiv \partial^2 Q / \partial (N^P c^P) \partial (N^P c^P)$, and define Q_{RR} and Q_{PR} accordingly. From our assumptions, $Q_P > 0$, $Q_{PP} < 0$, $Q_R < 0$, and $Q_{RR} > 0$.

A reasonable approximation to work with is that the probability Q is homogenous of degree zero in its arguments.[5] That is, $Q(N^P c^P, N^R c^R) = Q(\lambda N^P c^P, \lambda N^R c^R)$, for

[5] This can be modified to include the idea that the probability of a successful outcome is zero

any $\lambda > 0$.[6] This implies that Q remains unchanged if the spending by the poor and the rich change in the same proportion. In other words, the poor and the rich are "symmetrically" productive at creating and deterring class struggle. The homogeneity assumption implies that

$$Q_P / Q_R = -c^R / c^P. \tag{7}$$

The allocation of resources. Each person decides how much to spend on class struggle. In doing so, this individual takes as given the spending of his opponent class. The equilibrium spending of each person is a Nash equilibrium of this game. Formally, each player $h = P$ and R playing against $\ell = P$ and R with $h \neq \ell$ solves the following problem:

$$c^h(c^\ell) = \arg\max_{c^h \leq w^h} v^h(c^P, c^R). \tag{8}$$

The analysis presented in this paper is for interior solutions of problems such as (8), which entail positive values of c^P and c^R. Assuming an interior solution of (8), the second-order conditions are satisfied, and the optimality conditions are

$$Q_P g^P = 1, \text{ and } Q_R g^R = 1. \tag{9}$$

In turn, using (7),

$$c^P / c^R = -g^P / g^R. \tag{10}$$

Expression (9) permits us to compare, in equilibrium, the marginal impact of spending by the poor versus the rich on the probability of a successful outcome. For

unless the spendings by the poor and the rich exceed some minimum "threshold" levels. Doing so will leave most of our conclusions unchanged and will strengthen the others.

[6] Unless needed otherwise, we henceforth normalize the size of both groups to one. The effects of asymmetric group sizes are examined in a later part of the paper.

brevity in interpretations, we refer to Q_P as the "poor's marginal increase in the probability of a successful outcome" (due to a marginal increase in the spending by the poor). Analogously, we refer to $-Q_R$ as the "rich's marginal decrease in the probability of a successful outcome" (due to a marginal increase in the spending by the rich).

Proposition 1. *The poor's marginal increase in the probability of a successful outcome is larger than the rich's marginal decrease in the same probability. That is,*

$$Q_P > -Q_R. \tag{11}$$

The derivation and interpretation of (11) is intuitive. At the optimum, a poor person equates a dollar of his spending on class struggle to: (i) his marginal increase in the probability of a successful outcome, times (ii) his gain from a successful outcome, g^P. Likewise, a rich person equates a dollar of his spending on class struggle to: (i) his marginal decrease in the probability of a successful outcome, times (ii) the magnitude of his loss from a successful outcome, $-g^R$. These two relationships can be seen in (9). Next, recall that society loses some resources in the aftermath of a successful outcome. This means that the rich's loss from a successful outcome exceeds the poor's gain from the same. This, in turn, implies that the poor's marginal increase in the probability of a successful outcome is larger than the rich's marginal decrease in the same probability.

Proposition 2. *A poor person spends less on class struggle than a rich person. That is,*

$$c^P < c^R. \tag{12}$$

Recall that the rich's loss from a successful outcome exceeds the poor's gain. Next, note from (10) that the ratio of spendings of a poor person and a rich person is the same as the ratio of the gain to a poor from a successful outcome and the loss to a rich. Accordingly, expression (12) follows.

Some effects of class mobility. How does the level of class mobility m affect the spending on class struggle by the rich and the poor? How does mobility affect the overall probability of a successful class struggle? We now present some qualitative results on these questions. We use the notations: $c_m^h \equiv dc^h/dm$, for $h = P$ and R; $c_m \equiv dc/dm$; and $Q_m \equiv dQ/dm$.

Proposition 3. (a) *A poor person's spending on class struggle, as a ratio of a rich person's spending, is decreasing and concave in the level of class mobility.*[7] *That is,*

$$d(c^P/c^R)/dm < 0 \text{ and } d^2(c^P/c^R)/dm^2 < 0. \tag{13}$$

(b) *In a more mobile society, a poor person spends less on class struggle, and a rich person spends more, as a proportion of the total societal spending. That is,*

$$d[c^P/c]/dm < 0 \text{ and } d[c^R/c]/dm > 0. \tag{14}$$

The Appendix contains the derivation of (13). However, almost all of the conclusions in the preceding proposition can be understood as follows. Consider the first

[7] For additional interpretation of the result in the first part of (13), suppose hypothetically that $c_m^P < 0$, and $c_m^R < 0$. These inequalities hold in the special case presented later in the paper. Then, the result just noted implies that $|d \ln c^P/d \ln m| > |d \ln c^R/d \ln m|$. In other words, the magnitude of the elasticity of a poor person's spending with respect to class mobility is larger than the magnitude of the corresponding elasticity of a rich person's spending. On the other hand, the result under consideration does not imply that $|c_m^P| > |c_m^R|$. That is, it is not implied that the magnitude of the reduction in a poor person's spending, due to a small increase in class mobility, is larger than the magnitude of the corresponding reduction in a rich person's spending.

part of (13). For brevity, define $a \equiv c^P/c^R$. Then from (10), $a - 1 = -g/g^R$. Here, $g < 0$, and from (5) and (6) respectively, $g_m^R > 0$ and $g_m = 0$. Hence, $a - 1$ is decreasing in m, which implies that a is decreasing in m. Next, it is easily seen that the preceding result leads to (14). If a is decreasing in m, then $a + 1$ is also decreasing in m. Therefore, from the definition of a, the societal spending on class struggle, as a ratio of the spending by a rich person, is decreasing in m. Equivalently, a rich person's spending, as a ratio of the societal spending, is increasing in m. In turn, a poor person's spending, as a ratio of the societal spending, is decreasing in m.

The next proposition is derived in the Appendix.

Proposition 4. *In a more mobile society:* (a) *A poor person spends less on class struggle. That is,*

$$c_m^P < 0. \tag{15}$$

(b) *If the gain to the poor from a successful class struggle is larger than half of the magnitude of the loss to the rich, then the total societal spending is lowered. That is,*

$$g^P > -g^R/2 \Rightarrow c_m < 0. \tag{16}$$

One could think of the expenditure by the poor as being spent on activities that increase their societal power. In different situations these activities can take different forms. Among these are organizing strikes and protests, fostering unrest, seeking influence over the government's various distributive policies including those related to job reservations and quotas. Analogously, one could think of the expenditure by the rich as being spent on activities that diffuse the societal power of the poor. Expression (15) suggests that we should expect more mobile societies to experience fewer activities by the poor to increase their societal power, but not necessarily fewer activities by the rich to diffuse the preceding power. Expression (16) provides a sufficient, but not a necessary, condition for a more mobile society to have less societal spending on class struggle.

Proposition 5. *The probability of a successful class struggle is lower in a more mobile society. That is,*

$$Q_m < 0. \tag{17}$$

This result is a consequence of the first part of (13). Since Q is homogenous of degree zero in its arguments, we can define $q(c^P/c^R) \equiv Q(c^P, c^R)$, where q is increasing in its argument. Since the spending by a poor person, as a ratio of the spending by a rich person, is decreasing in m, it follows that the probability of a successful class struggle, q, is decreasing in m.

Proposition 5 can be interpreted as follows. In more mobile societies, the poor spend less on increasing their societal power. If the spending by the rich is held constant, the preceding effect decreases the probability of a successful class struggle. On the other hand, the rich may increase or decrease their spending. What this proposition shows is that, accounting for both the reactions of the poor and of the rich, a more mobile society will have a lower probability of a successful class struggle.

A special case. Consider the case in which the probability of a successful class struggle takes the form $Q \equiv c^P/(c^P + c^R)$, which is homogenous of degree zero. This yields the following closed-form solutions:

$$c^P = \frac{-g^R[g^P]^2}{[g^P - g^R]^2}, \quad c^R = \frac{g^P[g^R]^2}{[g^P - g^R]^2}, \tag{18}$$
$$c = \frac{g^R g^P}{g^R - g^P}, \text{ and } Q = \frac{g^P}{g^P - g^R}.$$

As one would expect, all of the results obtained within the preceding model hold in the present special case. In addition, $c_m^R < 0$ here, which means that $c_m < 0$, without needing any restriction on the gains to the poor and the rich. Another implication of the present case is that Q is not only decreasing in m, but it is also concave in m.

The expressions in (18) can be graphed for any feasible set of parameters. In the figures presented below, $\delta = 0.7$, $w^R/w^P = 8$, and $(w^P + w^R)/2w = 1.125$. That is, a rich person has eight times the income of a poor person, and twenty five percent of societal income is destroyed in the aftermath of a successful class struggle. Without loss of generality, we use the normalization $w = 1$. This yields $w^P = 0.25$, and $w^R = 2$. The range of values of m graphed below satisfies the restrictions that $g^P > 0$, $g^R < 0$ and $g < 0$.

[Figures 1 and 2 come about here]

Figure 1 illustrates the decrease in resources spent on class struggle by the poor and the rich, in relation to the level of class mobility. Figure 2 illustrates the decreasing and concave relationship between the probability of a successful class struggle and the level of class mobility. Also note that the spending by each of the two classes, as well as the probability of a successful outcome, become close to zero at the upper end of class mobility depicted in these figures. These and other properties of the graphs in Figures 1 and 2 are independent of the precise values of the parameters used in creating these graphs.

III. Some More General Models

In this section, we present some extensions of the model analyzed earlier. Among the issues considered are: (i) free-riding, (ii) unequal population sizes of the groups,

(iii) risk-aversion, and (iv) more than two generations. Unless noted otherwise, the assumptions made in the previous section are maintained in the treatment of each issue.

Free-riding. We consider a polar form of free-riding. In determining one's own spending, a person takes not only the resources spent by the opposite class as given, but also takes the resources spent by the other members of one's own class as given. We drop the normalization of the populations, and write Q as $Q(Nc^P, Nc^R)$, where $N^P = N^R = N$. The equilibrium is a Nash solution to the following allocation problem facing each individual of group h:

$$\begin{aligned} c^h &= \arg\max_{c^h \leq w^h} Y^h + Q(C^P, C^R)g^h, \\ \text{s.t.} \quad C^h &= (N-1)\bar{c}^h + c^h, \text{ with } \bar{c}^h \text{ given,} \\ C^\ell &= N^\ell \bar{c}^\ell, \text{ with } \bar{c}^\ell \text{given,} \\ &\quad \text{where } \ell \text{ and } h = P \text{ and } R, \text{ and } \ell \neq h. \end{aligned}$$

Assuming a symmetric interior equilibrium within each class, let $\tilde{c}^h$ denote the resources spent by a member of class h. It can be verified that all of the propositions presented in the last section hold with free-riding.

However, under free-riding, we would expect the resources spent by a member of either class to be smaller than those without free-riding. That is,

$$\tilde{c}^h < c^h \tag{19}$$

A sufficient condition for (19) is that the cross-effect Q_{PR} be negligible compared to Q_{PP} and Q_{RR}. Weaker sufficient conditions also lead to (19), as is shown in the Appendix.

Unequal populations of groups. Q is now written as $Q(N^P c^P, N^R c^R)$, where $N^P > N^R$; that is, the poor outnumber the rich. The societal gain from a successful struggle, as compared to an unsuccessful one, is now $N^P g^P + N^R g^R$. As before, we assume that the preceding gain is negative. The societal expenditure on class struggle is now $C \equiv N^P c^P + N^R c^R$. With this redefinition, it can be established that all of the propositions described in the previous section hold in the present case. Further, since Q is homogeneous of degree zero in its arguments, we can use (10) to write $q(-N^P g^P / N^R g^R) \equiv Q$, where q is increasing in its argument. It thus follows that *a larger population of the poor will raise the probability of a successful class struggle, whereas a larger population of the rich will lower this probability.*

Risk-aversion. So far we have assumed that individuals are risk-neutral. With risk-aversion, the net income streams are valued using an increasing and concave utility function $u(\cdot)$. Thus, the symbols in (1) and (2) get respectively redefined as $y^h \equiv u(w^h - c^h) + \delta u(w)$, and $Y^h \equiv u(w^h - c^h) + \delta[(1-m)u(w^h) + mu(w^\ell)]$. Using assumptions analogous to those used in Section 2, it turns out that all of the results presented in that section continue to hold.[8]

A multi-generational model. The model of the previous section can be extended to multi-generational settings. A brief illustration is as follows. Each of the poor and rich lives for one period, and has one offspring. A member of any generation can be in one of the following three states: (i) poor (P), (ii) rich (R), and (iii) "absorbed" (A). By "absorbed" we mean that a successful class struggle has taken place in some prior generation, and all income inequality has been removed. Notice that if

[8] The derivations of these results are straightforward but lengthy. Accordingly, these are available from the authors upon request.

one member of a generation is in the "absorbed" state, then other members are too. We use the subscript $t = 1, ..., T$ to index generations.

Define $v_t^h(c_t^P, c_t^R)$ as the expected utility of a member of generation t in state $h = P$ and R, where c_t^h represents his spending. Let v_t^A denote the utility of a member of generation t in state A. Once the economy is in the absorbed state A, each member of the subsequent generations has an income w. Hence,

$$v_t^A \equiv \sum_{i=0}^{T-t} \delta^i w = \frac{1 - \delta^{T-t+1}}{1 - \delta} w. \tag{20}$$

On the other hand, a member of generation $t < T$ finding himself in state $h = P$ and R, solves the following problem,

$$c_t^h(c_t^\ell) = \arg\max_{c_t^h \leq w^h} v_t^h(c_t^P, c_t^R), \text{ for } \ell = P \text{ and } R, \text{ and } \ell \neq h. \tag{21}$$

The function v_t^h appearing in the previous equation can be defined in the same way as in (3). That is,

$$v_t^h(c_t^P, c_t^R) \equiv Y_t^h + Q(c_t^P, c_t^R)\hat{g}_t^h, \text{ where} \tag{22}$$

$$\begin{aligned} Y_t^h &\equiv (w^h - c_t^h) + \delta\left[m v_{t+1}^\ell(c_{t+1}^P, c_{t+1}^R) + (1-m) v_{t+1}^h(c_{t+1}^P, c_{t+1}^R)\right], \\ y_t^h &\equiv (w^h - c_t^h) + \delta v_{t+1}^A, \text{ and} \\ \hat{g}_t^h &\equiv y_t^h - Y_t^h, \text{ for } h \text{ and } \ell = P \text{ and } R, \text{ and } \ell \neq h. \end{aligned} \tag{23}$$

Members of the last generation T consume their full income; that is, $v_T^h = w^h$ for $h = P$ and R, and $v_T^A = w$. As shown in the Appendix, the first-order condition for a member of generation $t < T$ in state $h = P$ and R is:

$$Q_h(c_t^P, c_t^R)\hat{g}_t^h = 1, \text{ for } h = P \text{ and } R. \tag{24}$$

These are the same first-order condition as (9) in the setting of two generations, except that g^h has been replaced by $\hat{g}_t^h$. It is thus apparent that the qualitative nature of the optimality here is similar to that analyzed in the last section.

IV. Concluding Remarks

The focus of this paper has been to explore some effects of class mobility on class struggle. We have done this in a highly parsimonious set of models, leaving out a number of considerations. These models can be extended and enriched in many ways. We believe that doing so would not alter the central message of this paper regarding the importance of class mobility in the context of class struggle.

For example, a class struggle can have many different outcomes, rather than only binary ones (i.e., success or failure) as we have depicted. Another issue is that an economy is affected in many different ways by a higher level of class mobility. These include a potentially more productive allocation of human inputs, with positive consequences on the economy's growth. This in turn may have an added effect of reducing the incentives for class struggle. On the other hand, the societal spending on class struggle has a likely negative effect on the economy's growth. Hence, low-growth "traps" might exist in a society in which the current class mobility is low, and a large part of the society's resources are spent on class struggle, such that the growth prospects for some of the future generations are reduced, preventing them from escaping a low-growth trap.

Appendix

Derivation of expression (13). Define $a \equiv c^P/c^R$. >From (10), $a = -g^P/g^R$. Thus,

$$da/dm = -(g^R g^P_m - g^P g^R_m)/(g^R)^2 = g^R_m g/(g^R)^2 < 0. \tag{A1}$$

Here we have used the facts that $g^R_m = -g^P_m > 0$, and $g < 0$. Next, recall: (i) from (4) that $g^R < 0$, (ii) from (5) that $\partial g^R_m/\partial m = 0$, and (iii) from (6) that $g_m = 0$. Hence, (A1) yields $d^2a/dm^2 = -2(g^R_m)^2 g/(g^R)^3 < 0$.

Proof of Proposition 4. Let $k^h \equiv g^h_m/g^h$ for $h = P$ and R. It is seen from (4) and (5) that $k^h < 0$ for $h = P$ and R. Further, from (6), $g^R_m = -g^P_m$. Hence, using $g < 0$ it follows that

$$\rho \equiv k^P/k^R = -g^R/g^P > 1. \tag{A2}$$

Differentiating the first-order conditions (9), and rearranging, we obtain

$$c^P_m = -(1/D)\{Q_{RR}Q_P k^P - Q_{PR}Q_R k^R\}, \text{ and} \tag{A3}$$

$$c^R_m = -(1/D)\{Q_{PP}Q_R k^R - Q_{PR}Q_P k^P\}. \tag{A4}$$

where $D \equiv Q_{RR}Q_{PP} - Q^2_{PR} < 0$. The functions Q_P and Q_R are homogenous of degree minus one. Hence, the Euler's theorem and (7) yield

$$Q_{PR} = -Q_P/c^R + Q_{PP}Q_R/Q_P, \text{ and} \tag{A5}$$

$$Q_{PR} = -Q_R/c^P + Q_{RR}Q_P/Q_R. \tag{A6}$$

Substitution of (A5), (A6) and (A2) into (A3) and (A4), and a rearrangement of the resulting expressions, yields

$$c^P_m = -(k^R/D)\left\{Q_{RR}Q_P(\rho - 1) + Q^2_R/c^P\right\} < 0, \text{ and} \tag{A7}$$

$$c^R_m = -(k^R/D)\left\{\rho Q^2_P/c^R - Q_{PP}Q_R(\rho - 1)\right\}. \tag{A8}$$

The sign for c_m^P is obtained by using $k^R < 0$, $D < 0$, $Q_{RR} > 0$, $Q_P > 0$, $\rho > 1$, and $c^P > 0$. This proves part (a) of the proposition.

Next, $c_m = c_m^P + c_m^R$. From (A7) and (A8),

$$c_m = -(k^R/D)\{Q_R^2/c^P + \rho Q_P^2/c^R + (\rho - 1)[Q_P Q_{RR} - Q_R Q_{PP}]\}. \tag{A9}$$

Combining equations (A5) and (A6), we get

$$Q_R Q_{PP} = -Q_P Q_R/c^P - \rho Q_P Q_{RR} + Q_P^2/c^R. \tag{A10}$$

Substituting (A10) into (A9), using the fact that $\rho = c^R/c^P = -Q_P/Q_R$, and rearranging the resulting expression, we obtain,

$$c_m = -\frac{k^R}{D}\{(\rho - 1)(1 + \rho)Q_{RR}Q_P\} - \frac{k^R}{D}\frac{Q_R^2}{c^R}\rho(1 + 2\rho - \rho^2). \tag{A11}$$

The first term of equation (A11) is negative since $k^R < 0$, $D < 0$, $\rho > 1$, $Q_{RR} > 0$, and $Q_P > 0$. A sufficient condition for $c_m < 0$ is therefore $\phi \equiv 1 + 2\rho - \rho^2 > 0$. Note that $\partial\phi/\partial\rho = -2(\rho - 1) < 0$. Thus, there exists a unique ρ^* (which is approximately 2.41), such that $c_m < 0$ for all $\rho < \rho^*$. Hence a sufficient condition for $c_m < 0$ is $\rho \leq 2$. Since $\rho \equiv -g^R/g^P$, we can restate the condition $\rho \leq 2$ as $g^P \geq -g^R/2$.

Conditions for (19). With free-riding, the first-order conditions for a poor and a rich are respectively

$$Q_P(C^P, C^R)g^P = 1, \text{ and } Q_R(C^P, C^R)g^R = 1 \tag{A12}$$

For comparison, the corresponding first-order conditions without free-riding are

$$Q_P(C^P, C^R)Ng^P = 1, \text{ and } Q_R(C^P, C^R)Ng^R = 1 \tag{A13}$$

Consider the set of equations

$$Q_P(C^P, C^R)Zg^P = 1, \text{ and } Q_R(C^P, C^R)Zg^R = 1 \tag{A14}$$

If $Z = 1$, then (A14) becomes (A12) with $C^h = N\tilde{c}^h$. On the other hand, if $Z = N$, then (A14) becomes (A13) with $C^h = Nc^h$. Thus, to derive sufficient conditions for (19), we perturb (A14) with respect to Z, at $Z = N$. This yields

$$\frac{\partial c^P}{\partial Z} = \frac{(Q_R Q_{PR} - Q_P Q_{RR})}{NZD}, \text{ and} \tag{A15}$$

$$\frac{\partial c^R}{\partial Z} = \frac{(Q_P Q_{PR} - Q_R Q_{PP})}{NZD}, \tag{A16}$$

where recall that $D < 0$. If Q_{PR} is negligible compared to Q_{PP} and Q_{RR}, then both (A15) and (A16) are positive. Hence, (19) holds. Otherwise, if $Q_{PR} > 0$, a sufficient condition for (19) is $Q_{PR} < (Q_R/Q_P)Q_{PP}$. On the other hand, if $Q_{PR} < 0$, a sufficient condition for (19) is $Q_{PR} < (Q_P/Q_R)Q_{RR}$.

Derivation of expression (24). The first-order condition for a member of generation $t < T$ in state $h = P$ and R is

$$\begin{aligned} \frac{\partial v_t^h(c_t^P, c_t^R)}{\partial c_t^h} = 0 \Leftrightarrow -1 \\ + \delta(1 - Q(c_t^P, c_t^R)) \left[m \frac{\partial v_{t+1}^\ell}{\partial c_{t+1}^\ell} \frac{\partial c_{t+1}^\ell}{\partial c_t^h} + (1-m) \frac{\partial v_{t+1}^h}{\partial c_{t+1}^h} \frac{\partial c_{t+1}^h}{\partial c_t^h} \right] \\ + \frac{\partial Q(c_t^P, c_t^R)}{\partial c_t^h} \hat{g}_t^h = 0, \text{ for } \ell = P \text{ and } R, \text{ and } \ell \neq h. \end{aligned} \tag{A17}$$

This first-order condition holds for all dates $t < T$. In particular, it holds for date $t + 1$. Thus, $\partial v_{t+1}^h(c_{t+1}^P, c_{t+1}^R)/\partial c_{t+1}^h = 0$, for $h = P$ and R. Substitution of the preceding expression into (A17) yields (24).

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